

University of Montana

ScholarWorks at University of Montana

Graduate Student Theses, Dissertations, &
Professional Papers

Graduate School

1982

Econometric estimation of demand and supply curves for timber in Montana, 1962-1980

Gerard A. Majerus

The University of Montana

Follow this and additional works at: <https://scholarworks.umt.edu/etd>

Let us know how access to this document benefits you.

Recommended Citation

Majerus, Gerard A., "Econometric estimation of demand and supply curves for timber in Montana, 1962-1980" (1982). *Graduate Student Theses, Dissertations, & Professional Papers*. 2132.
<https://scholarworks.umt.edu/etd/2132>

This Thesis is brought to you for free and open access by the Graduate School at ScholarWorks at University of Montana. It has been accepted for inclusion in Graduate Student Theses, Dissertations, & Professional Papers by an authorized administrator of ScholarWorks at University of Montana. For more information, please contact scholarworks@mso.umt.edu.

COPYRIGHT ACT OF 1976

THIS IS AN UNPUBLISHED MANUSCRIPT IN WHICH COPYRIGHT SUBSISTS. ANY FURTHER REPRINTING OF ITS CONTENTS MUST BE APPROVED BY THE AUTHOR.

MANSFIELD LIBRARY
UNIVERSITY OF MONTANA
DATE: 1982

ECONOMETRIC ESTIMATION OF DEMAND
AND SUPPLY CURVES FOR TIMBER IN
MONTANA, 1962-1980

By

GERARD A. MAJERUS

B.S.F., University of Montana, 1980

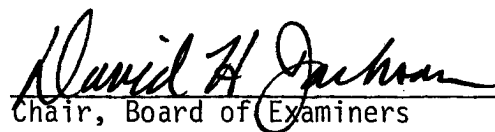
Presented in partial fulfillment of the
requirements for the degree of

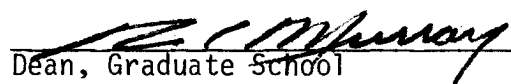
Master of Science in Forestry

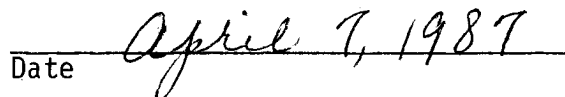
UNIVERSITY OF MONTANA

1982

Approved by:


Chair, Board of Examiners


Dean, Graduate School


Date

UMI Number: EP34506

All rights reserved

INFORMATION TO ALL USERS

The quality of this reproduction is dependent upon the quality of the copy submitted.

In the unlikely event that the author did not send a complete manuscript and there are missing pages, these will be noted. Also, if material had to be removed, a note will indicate the deletion.



UMI EP34506

Published by ProQuest LLC (2012). Copyright in the Dissertation held by the Author.

Microform Edition © ProQuest LLC.

All rights reserved. This work is protected against unauthorized copying under Title 17, United States Code



ProQuest LLC.
789 East Eisenhower Parkway
P.O. Box 1346
Ann Arbor, MI 48106 - 1346

Majerus, Gerard A., M.S., December, 1982

Forestry

Econometric Estimation of Demand and Supply Curves for Timber in Montana, 1962-1980 (52 pp.)

Director: David H. Jackson

An econometric model for the demand and supply of timber in Montana for the time period 1962 through 1980 is estimated using two-stage least squares. With these equations Jackson's (1981) analytical method is used to estimate the demand equations for individual national forests in Montana. A method is presented for determining the error associated with the analytically derived demand equations and is applied to the equations for the national forests in Montana.

The results indicate that the price elasticity of demand for timber in Montana is inelastic while for the individual national forests it is price elastic. The error for an analytically derived demand equation is dependent on the variance of the Montana demand equation, the variance for the non-national forest's supply and the covariance between the two.

ACKNOWLEDGEMENTS

This study was made possible by the preliminary research and advice from my committee chairman Dr. David H. Jackson and the review and financial assistance provided by Kent P. Connaughton. Also, this study would not have been completed without the continuous encouragement and understanding from Yvette Ellen.

Thanks is also extended to the members of my committee: Dr. Richard Shannon, Dr. Michael Kupilik, James Faurot and Dr. Alan McQuillan. Appreciation is also extended to Terry Raettig and Charles Fudge for their assistance and cooperation in the data collection and analysis.

TABLE OF CONTENTS

	Page
ABSTRACT	ii
ACKNOWLEDGEMENTS	iii
LIST OF TABLES, FIGURES AND GRAPHS	v
 CHAPTER	
1. INTRODUCTION.	1
2. LITERATURE REVIEW	3
3. PROCEDURES.	10
4. RESULTS	16
5. DISCUSSION.	30
 APPENDICES	
A. INPUT DEMAND FUNCTION	34
B. DATA USED IN THE DEMAND AND SUPPLY MODEL.	36
C. VARIANCE OF A NATIONAL FOREST'S DEMAND EQUATION .	43
D. IDENTIFICATION OF THE DEMAND AND SUPPLY MODEL . .	45
E. CORRELATION COEFFICIENTS AND THE FARRAR-GLAUBER TEST FOR THE PRESENCE OF MULTICOLLINEARITY. . . .	48
F. CALCULATION OF DERIVED DEMAND EQUATIONS	50
SELECTED BIBLIOGRAPHY.	51

LIST OF TABLES, FIGURES AND GRAPHS

TABLE	Page
1. Montana Demand Equation	17
2. Timber Demand and Supply Elasticities for the Montana Demand and Supply Model	20
3. Montana Supply Equation	21
4. Derived Demand Equations for Individual National Forests in Montana and Standard Errors.	24
5. Mean Market Shares, Percent Standard Errors and Price Elasticities of Demand for National Forests in Montana (computed at the sample period means).	25
6. Derived Demand Equations for the Eastside and West- side National Forests and the National Forest System in Montana.	28
7. Mean Market Shares, Percent Standard Errors and Price Elasticities of Demand for the Eastside Forests, Westside Forests and National Forest System in Montana (computed at the sample period means).	29
8. Montana Data.	36
9. Correlation Coefficients.	48

FIGURE

1. Market Demand, Non-national Forest's Supply and National Forest Demand.	4
2. Regional Demand, Non-national Forest's Supply and National Forest Demand.	7
3. Market Demand and Supply and National Forest Demand.	8
4. Historical Lumber and Wood Products Price Index, Logging, Hauling and Manufacturing Costs and Stumpage Price.	19
5. Predicted and Observed Values; Montana Demand and Supply Model.	23

LIST OF TABLES, FIGURES AND GRAPHS (cont)

GRAPH	Page
1. Price of Stumpage and Harvest	39
2. Harvest and Relative Lumber and Wood Products Price Index	40
3. Harvest and Logging, Hauling and Manufacturing Costs	41
4. Harvest and Stock	42

Chapter 1

INTRODUCTION

The current land allocation and harvest scheduling model used by the Forest Service allows the specification of a demand schedule for timber. The present Forest Service practice is to assume that a National Forest is a price taker and bases policy decisions on investment analysis using a horizontal demand curve. The Forest Service assumes the timber output of the Lolo National Forest will all be sold and would have no impact on selling price (Lolo Draft EIS 1980). The price elasticity of demand is assumed to be perfectly elastic. If this assumption is violated, the results obtained can be invalid (Hrubes et al. 1976).

Recent attention has focused on using downward sloping demand curves for timber in National Forest planning. The Forest Service may influence the price of stumpage through the amount of inventory that is sold or withheld from the market (Walker 1980). Specification of a downward sloping demand curve would make investment return calculations, price effects and harvest scheduling more meaningful (Jackson 1980).

Previous studies have centered on regional demand and supply relationships (Adams et al. 1980), subregional demand and supply relationships (Jackson 1981) and specific market areas (Walker 1980).¹

¹ A regional demand and supply relationship is for a number of States while a subregional relationship is for an individual State.

These earlier studies did not review the errors associated with estimating the demand and supply for timber.

The objective of this study is to develop a demand and supply model for timber in Montana for the time period 1962 through 1980 and analytically derive the demand equations for each National Forest in Montana.² In addition, a method for determining the error associated with an analytically derived demand equation is developed. An analysis of the errors will give insight into the accuracy and use of demand curves in forest planning. This study is concerned with the stumpage market in Montana and the influence the Forest Service has on stumpage price.

² This study was to include northern Idaho as well but due to data limitations northern Idaho was excluded from the analysis.

Chapter 2

LITERATURE REVIEW

Timber is an input in the production process of wood products, therefore, the demand for timber is derived from the consumer's demand for the final output. The demand for an input is a derived demand or an input demand function (Russel et al. 1979). Various analytical methods are used to estimate the input demand function for stumpage. The best approach would be to estimate the derived demand from an empirical production function. Indirect estimation techniques are used to link the stumpage market with the end product market due to data limitations and modeling mis-specifications in estimating the production function for the wood products industry. Three recent approaches were derived by Walker (1980), Haynes, Connaughton and Adams (1981) and Jackson (1981). Following is a brief review of their methodologies.

Walker's approach focuses on two points (A, A^1) to estimate the demand curve for a market area. Point "A" is the equilibrium market price and quantity while " A^1 " is the current industrial mill capacity at a zero price. When an industry is operating at mill capacity the price paid for additional stumpage is zero. A line connecting these two points is the demand curve for a market area (see Figure 1). To derive the demand curve for an individual National Forest you must know the non-national forest's supply curve. The supply curve is estimated with two points (B, B^1). Point "B" is the equilibrium market price

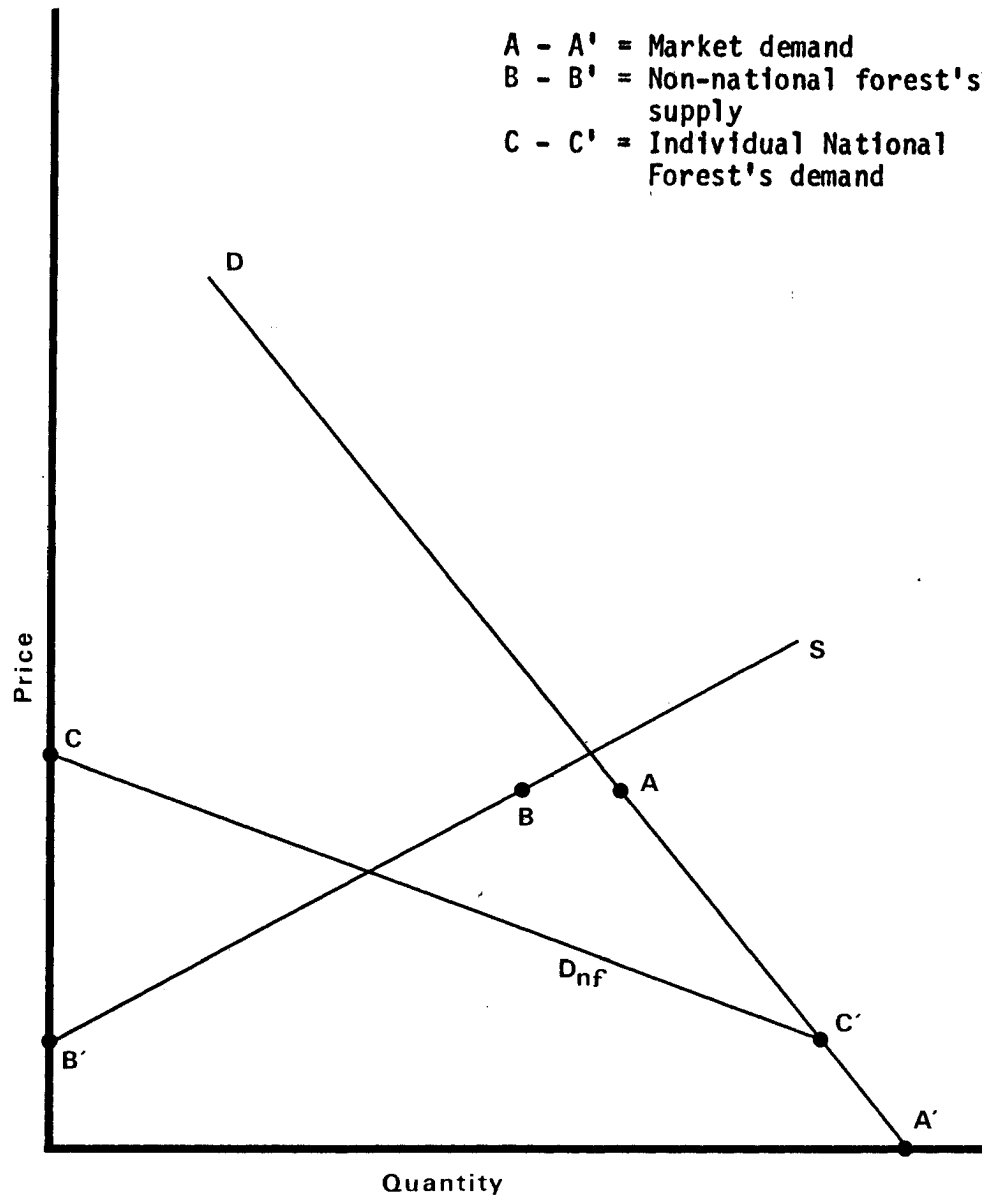


Figure 1.--Market demand, non-national forest's supply and National Forest demand.

Source: Walker, John L. 1980. An analysis of timber demand in the Lolo National Forest market area. Unpublished. 32p.

and non-national forest's quantity while " B^1 " is a non-zero price corresponding to a supply of zero. The non-zero price is equal to the logging and hauling costs when the stumpage price is zero (Connaughton 1981). A line connecting points "B" and " B^1 " depicts the non-national forest's supply curve (see Figure 1). The two points (C, C^1) are necessary to estimate the derived demand curve for an individual National Forest. Point "C" is the price where the non-national forest's supply and market demand curves intersect and a quantity of zero. The other point, " C^1 ", is the price when the non-national forest's supply is zero and the market demand quantity at that price. A line connecting these two points is the derived demand curve for an individual National Forest (see Figure 1).

With the Walker approach no linkage is made between the product market and stumpage market, except through mill capacity, and no explanation is given for the effects logging and manufacturing costs have on the demand for timber (Connaughton 1981). This method does not consider factors which cause shifts in the demand and supply relationship and there is no estimate of the error associated with either the market demand curve or an individual forest demand curve (Jackson 1981). Shifts in the demand curve will only occur with changes in mill capacity and real stumpage price. For planning purposes some estimate of mill capacity is necessary. In the short run, mill capacity will remain relatively constant, but in the long run the capacity will change with varying market conditions. There is no reason to believe that the price paid for timber is zero at mill

capacity (Connaughton 1981). Walker does not expect that stumpage would be sold at a zero price, however, mill capacity determines the slope of the market demand curve. Mill capacity is not a desirable predetermined variable because it defines the slope of the demand curve.

Adams and Haynes (1980) estimated demand and supply relationships for the 1980 Renewable Resources Planning assessment. The demand for timber was determined from the supply of wood products. Subsequently, Haynes, Connaughton and Adams (1981) used these stumpage price and quantity projections to estimate the demand curves for various regions in the United States. This was based on the assumption that timber supplied from the National Forests is not price responsive while it is for all other ownerships. The demand curve for the National Forests is derived by subtracting the non-national forest's supply from the regional demand curve for all ownerships. This method yields a demand curve that is more price elastic than the regional demand curve (see Figure 2).

Jackson (1980) estimated timber demand and supply equations for Montana with an econometric model. A method is presented whereby a demand curve for a National Forest can be estimated from the large area demand and supply equations and from that forest's share of the market. The demand curve for a National Forest is calculated by subtracting a fixed proportion of the total supply from the large area demand curve (see Figure 3). The proportion is determined by that forest's share of the market. This approach does not assume a fixed supply of

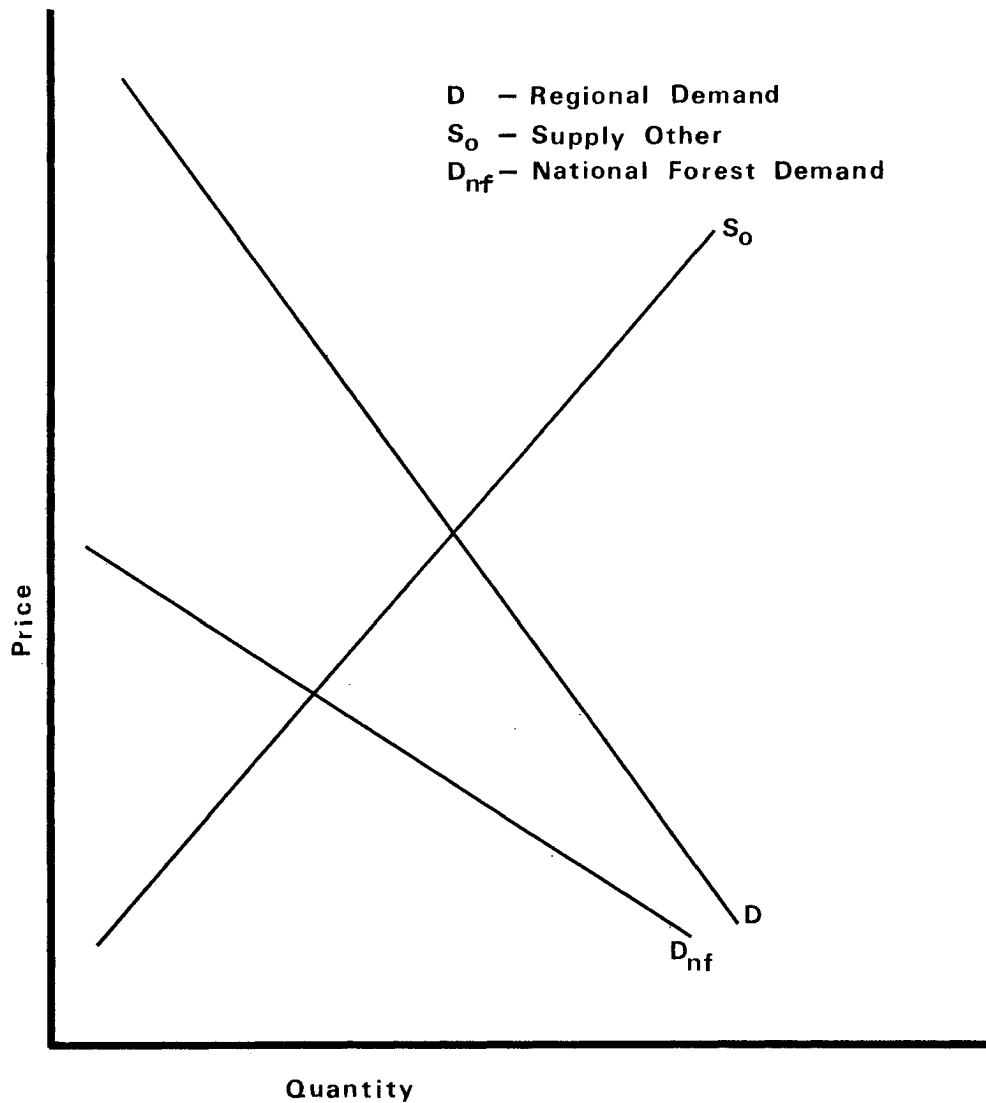


Figure 2.--Regional demand, non-national forest's supply and National Forest demand.

Source: Haynes, Richard W., Kent P. Connaughton and Darius M. Adams. 1981. Projections of the demand for National Forest stumpage by Region; 1980-2030. USDA For. Serv. Res. Paper PNW-282. Pac. Northwest For. and Range Exp. Stn., Portland, OR. 13p.

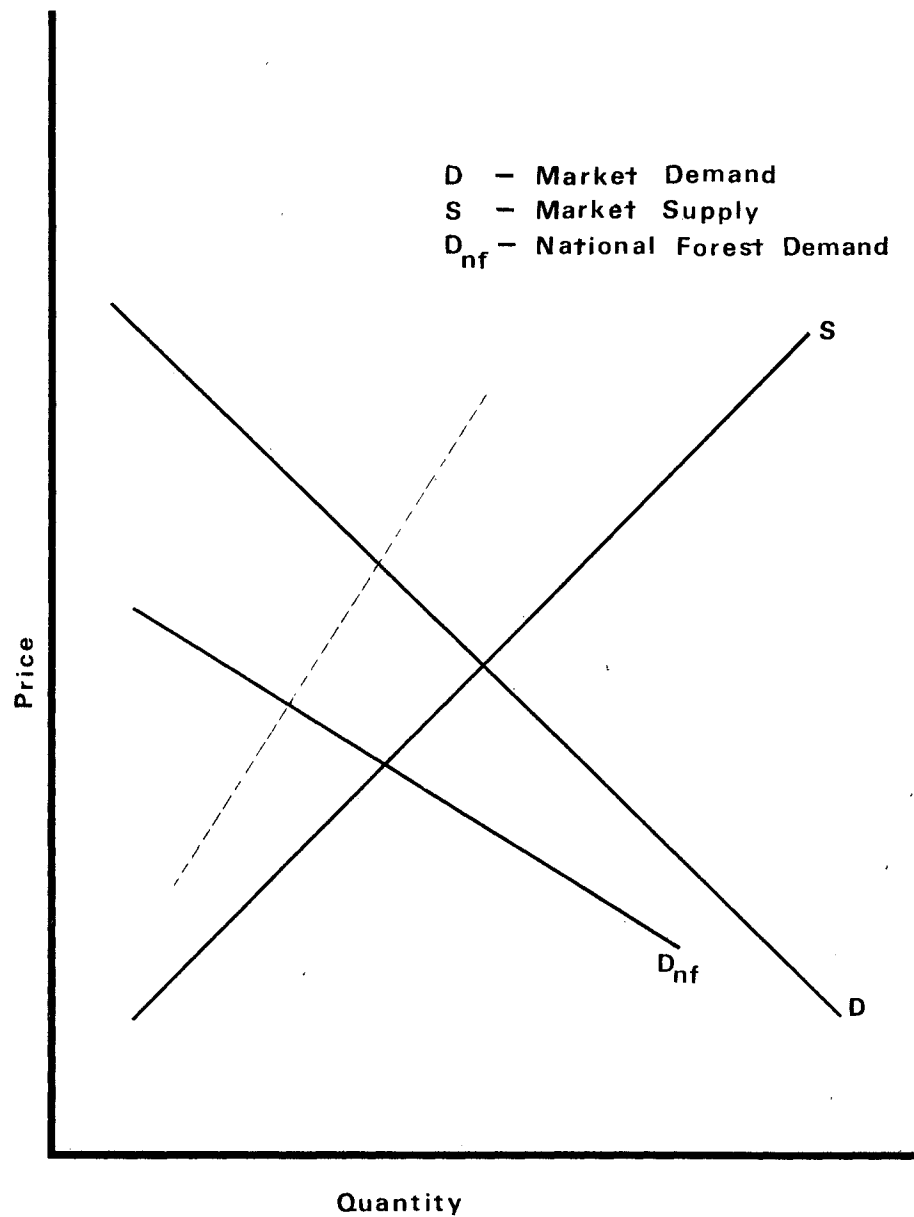


Figure 3.--Market demand and supply and National Forest demand.
Source: Jackson, David H. 1981. Sub-regional timber demand
analysis: remarks and an approach for prediction. Unpublished.
14p.

National Forest timber in the derivation of a derived demand curve for a smaller area.

Chapter 3

PROCEDURES

A demand and supply model is a system of equations describing the joint dependence of the quantity supplied and demanded as a function of price. The demand for and supply of a good are the various quantities per unit of time that will be taken off or placed on the market at all possible prices, other factors remaining constant (Leftwich 1979). Other exogenous variables are included in the model to explain shifts in the demand and supply relationship. Shifts in demand are a function of tastes and preferences, income, expectations, prices of related goods, number of consumers and availability. Shifts in supply are a function of the prices of inputs in the production process and the available production facilities. Simultaneous equation bias may occur if either the demand or supply curves are estimated directly. This bias can be eliminated by using two-stage least squares. Two-stage least squares will yield unbiased and consistent estimates of the parameters with a large sample size.

The demand and supply model in this analysis was developed from economic theory, previous analytical work (Schreuder et al. 1976, Adams et al. 1980 and Jackson 1981) and theoretical relationships for the supply and demand for timber described by Jackson (1981). The input demand function for timber depends on the price of stumpage, price of the end product and prices of substitutes and complements. For the derivation of the input demand function see Appendix A. The

demand function describes the derived demand for timber using two markets, the stumpage and product markets. The stumpage market will respond to changes in the price received for the end product.

The value of the end product less the amount paid for logging and manufacturing will be available for the purchase of timber (Connaughton 1981). Logging and manufacturing costs are the price of substitutes in the production process and hauling cost reflects the location of the timber from the mill. As the distance from the mill increases the price received for stumpage declines. The demand and supply model is as follows:

$$Qd_t = f(Ps_t, Pf_t, Pc_t) \quad (1)$$

$$Qs_t = f(Ps_t, S_t) \quad (2)$$

$$Qd_t = Qs_t \quad (3)$$

where Qd_t = quantity demanded in time t , m.b.f./yr (log scale)

Qs_t = quantity supplied in time t , m.b.f./yr (log scale)

Ps_t = stumpage price, mean annual National Forest cut value
in time t , \$/m.b.f. (log scale)

Pf_t = price of the end product, lumber and wood products
price index in time t

Pc_t = hauling, logging and manufacturing costs in time t ,
\$/m.b.f. (log scale)

S_t = inventory or stock, volume of sawtimber on commercial
timberland in time t , mm.b.f. (log scale)

t = time period 1962 through 1980, year

The hypotheses for the demand function are as follows:

$$\frac{\partial Q_d}{\partial P_s} < 0, \quad \frac{\partial Q_d}{\partial P_f} > 0, \quad \frac{\partial Q_d}{\partial P_c} < 0.$$

The demand curve is hypothesized to be negatively sloped and will shift with changes in the price of the end product and the price of substitutes. Higher wood product prices cause the demand curve to shift to the right while higher conversion costs will shift it to the left. The hypotheses for the supply function are as follows:

$$\frac{\partial Q_s}{\partial P_s} > 0, \quad \frac{\partial Q_s}{\partial S} > 0.$$

The supply curve is positively sloped and will shift to the left with less sawtimber available for harvest.

The above model was developed for Montana for the calendar years 1962 through 1980 inclusive (19 observations). All variables are annual values and all dollar values are adjusted to the base year 1967 using the Implicit Price Deflator for the Gross National Product. The price of stumpage is the National Forest cut value in dollars per m.b.f. The price of the end product is the relative lumber and wood products price index (1967=100) adjusted using the GNP Implicit Price Deflator. Costs are logging, hauling and manufacturing costs, dollars per m.b.f., for sales greater than two thousand dollars as recorded on the Forest Service timber sale appraisal forms. The stock data were interpolated linearly for the years 1962 through 1977 and extrapolated linearly for the years 1978 through 1980 using pub-

lished inventory values for 1952, 1962, 1970 and 1977.¹ See Appendix B for the data used in the demand and supply model.

Jackson's (1981) method was used to disaggregate the large area demand and supply relationship to the individual National Forests. The procedure is based on a National Forest's share of the market. The measure of the market share is the proportion of a National Forest's cut to the total cut in Montana. The market share is expressed as follows:

$$k = \frac{S}{T} \quad (4)$$

where S = an individual National Forest's cut (m.b.f.)
 T = the total cut in Montana (m.b.f.)
 k = the market share.

The derived demand relationship is expressed as follows:

$$q_{dd_t} = Qd_t - (1-k_t)Qs_t \quad (5)$$

where q_{dd_t} = derived demand for an individual National Forest's timber in time t , m.b.f./yr
 Qd_t = large area demand equation
 Qs_t = large area supply equation
 k_t = an individual National Forest's market share.

¹ Source: U.S.D.A. Forest Service. 1973. The outlook for timber in the United States. Forest Service Rept. No. 20. 367 p.
 U.S.D.A. Forest Service. 1978. Forest statistics of the U.S., 1977. Review draft, Washington, D.C. 133 p.

As the market share approaches one, a National Forest's demand curve converges to the large area demand curve (Jackson 1981). As Jackson pointed out earlier, the error properties of a National Forest's derived demand curve depends on the errors associated with the large area demand and supply equations.

The estimated variance of an individual National Forest's demand equation assuming a constant market share is as follows:

$$\text{Var}(q_{dd}) = \text{Var}(Q_d) + (1-k)\text{Var}(Q_s) - 2(1-k)\text{Cov}(Q_d, Q_s) \quad (6)$$

(Jackson 1981).

As the market share increases the variance will decrease. The variance depends on the market share given the large area demand and supply relationship. This variance is a combination of the variances and covariance of the large area demand and supply equations.

Since the market share is not a constant but changes over time, the variance of a National Forest's demand equation will depend on the market share. If the market share is a random variable, then the variance is as follows:

$$\text{Var}(q_{dd}) = \text{Var}(Q_d) + \text{Var}(mQ_s) - 2 \text{Cov}(Q_d, mQ_s) \quad (7)$$

where $m = (1-k)$ or the market share of all other ownerships

mQ_s = non-national forest's supply.

This formula is derived from the model for an individual National Forest's demand equation and the definition of variance (see Appendix C). Given the large area demand and supply relationship, the variance

depends on the variance of the demand equation, the variance of the non-national forest's supply and the covariance between the two.

Chapter 4

RESULTS

Two-stage least squares was used to estimate the coefficients of the large area supply and demand model. This method is appropriate when an equation is overidentified and will result in identical estimates as indirect least squares when an equation is exactly identified. The demand function is exactly identified and the supply function is overidentified (see Appendix D).

The results for the demand equation are given in Table 1. A one tailed t-test at the 90 percent confidence level was used to test the hypotheses and the signs of the coefficients. The critical value for t at the 90 percent confidence level for 15 degrees of freedom is 1.341. All the independent variables are statistically significant at this level and the hypotheses are correct. The critical value for the F statistic at the 95 percent confidence level is 3.29, therefore, the equation is statistically significant. The upper and lower limits for the Durbin-Watson statistic at the one percent level of significance are 0.74 and 1.41 respectively. Since the empirical value is greater than the upper limit we conclude there is no first order serial correlation.

The correlation between the independent variables ranges from 0.45 to 0.85 (see Appendix E). Since the correlation is high, the Farrar-Glauber test for multicollinearity was used. To test for the overall degree of multicollinearity the chi-squared statistic, χ^2 ,

Table 1. Montana Demand Equation.

<u>Variable Name</u>	<u>Coefficient</u>	<u>t-Statistic</u>
Stumpage price, National Forest cut value, \$/m.b.f.	- 7,607.53	-1.349
Relative lumber and wood products price index (1967=100)	7,263.74	1.807
Hauling, logging and manufacturing costs, \$/m.b.f.	- 5,574.23	-3.344
Constant	919,521.00	3.139
Overall F = 5.03		
\bar{R}^2 = .40		
Durbin-Watson = 1.56		
Standard error = 89,027		
% Standard error = 7.6		
Number of observations = 19		

was computed and found equal to 34.94. Since the empirical value is greater than the theoretical value at the 90 percent confidence level, $\chi^2_{.10} = 6.25$, we conclude that there is multicollinearity in the demand equation. One would expect some degree of intercorrelation between the independent variables due to the interdependence between the price of stumpage, price of end products and conversion costs. Because these variables change in the same direction over time, it becomes difficult to separate their influences on the dependent variable (see Figure 4). While conversion costs do not show the same fluctuating pattern as the price of stumpage and the price of the end product they do indicate the upward trend. There is no conclusive evidence concerning the degree of multicollinearity and how seriously it will affect the parameter estimates (Koutsoyiannis 1979), therefore, no correction procedure was applied.

A natural logarithmic model was used to obtain the elasticities of demand with respect to the independent variables. Taking the natural log of all the variables and then applying two-stage least squares yields the coefficients of elasticities with respect to the independent variables. See Table 2 for the elasticities of demand. The price elasticity of demand is -0.05. The market demand curve for timber in Montana is inelastic. In the short run, the demand curve for the stumpage market is thought to be inelastic (Hamilton 1970). There are several factors that affect the price elasticity of demand for an input. They are the substitution between factors of production, the price elasticity of the end product and the relative cost of the input as

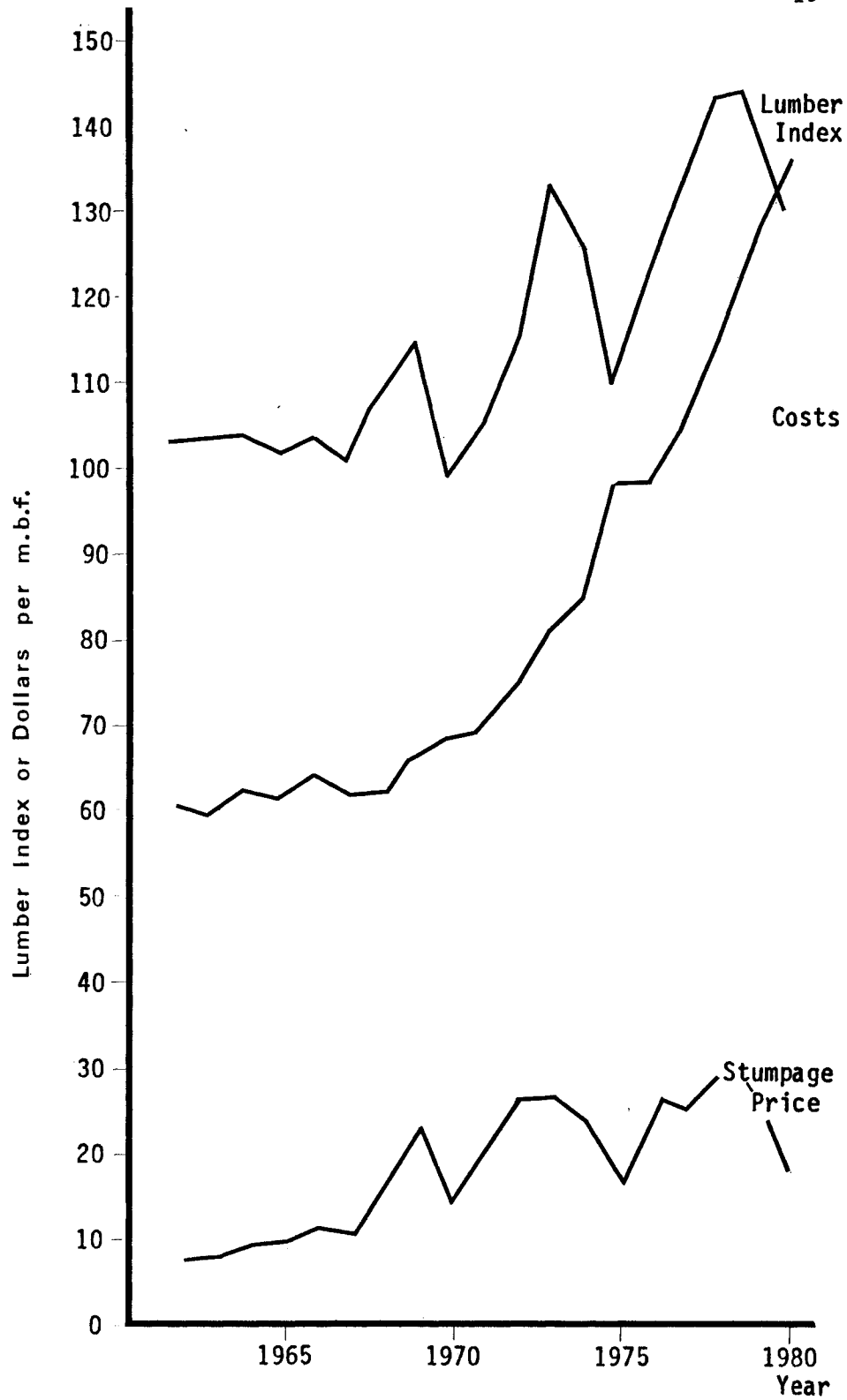


Figure 4.--Historical lumber and wood products price index, logging, hauling and manufacturing costs and stumpage price.

Table 2. Timber demand and supply elasticities for the Montana demand and supply model.

Demand elasticities with respect to--		
Stumpage price	End product price	Logging, hauling and mfg costs
-0.05	0.60	-0.42

Supply elasticities with respect to--	
Stumpage price	Stock-inventory
0.12	2.21

compared to total costs of production. Whenever substitution between factors of production is possible, the greater the substitution, the more price elastic the derived demand. There is limited substitution between timber and other factors of production in the manufacture of wood products, therefore, the price elasticity should be highly inelastic. The more elastic the demand for a commodity, the more elastic the derived demand for an input in the production process of that product. Adams and Haynes (1980) estimated demand elasticities in the Rocky Mountain Region for lumber and plywood to be -0.40 and -0.20 respectively. Therefore, one would expect the market demand

curve for timber to be inelastic. The greater the ratio of the cost of a factor of production is to total costs, the higher the elasticity of derived demand. The relative cost of timber in the production of wood products is low when compared to the total costs of production, thus, the elasticity would be inelastic (Mead 1966).

The results for the supply equation are as follows:

Table 3. Montana Supply Equation.

<u>Variable Name</u>	<u>Coefficient</u>	<u>t-Statistic</u>
Stumpage price National Forest cut value, \$/m.b.f.	8,251.79	1.866
Inventory-Stock, mm.b.f.	24.5433	4.012
Constant	-1,441,530	-2.115
Overall F	=	12.01
\bar{R}^2	=	.55
Durbin-Watson	=	1.50
Standard error	=	77,218
% Standard error	=	6.6
Number of observations	=	19

The critical value for t at the 90 percent confidence level for 16 degrees of freedom is 1.337. Both independent variables are statistically significant at this level and the signs of the coefficients are correct. The equation is statistically significant at the 95 percent

level: the critical value for F at this level is 3.63. There is no first order serial correlation since the Durbin-Watson statistic is greater than the upper limit at the one percent level of significance. The Farrar-Glauber test was used to test for multicollinearity. The $*\chi^2$ was computed and found equal to 11.3 (see Appendix E). Since the empirical value is greater than the theoretical value, $\chi^2_{.10} = 2.71$, we accept that there is significant multicollinearity in the supply function. The demand and supply equations are solved simultaneously to determine the equilibrium price and quantity. For a comparison of the predicted and observed values see Figure 5. The elasticities for the supply equation were derived in the same manner as the elasticities for the demand equation. See Table 2 for the elasticities of supply with respect to price and stock.

The individual National Forest demand equations were analytically derived using Jackson's (1981) model (see Appendix F). The results for each National Forest in Montana are given in Table 4 along with the standard errors. The standard errors were calculated using the formula developed earlier. As the market share decreases the slope of a derived demand curve becomes more horizontal or the stumpage price coefficient becomes greater. The smaller the market share the less impact changes in the harvest level have on price. With lower levels of the market share the variance becomes larger. The percent standard errors range from 30 percent for the Kootenai National Forest to 2,098 percent for the Custer National Forest. The mean market shares for the Kootenai and Custer are 0.1552 and 0.0016 respectively (see Table 5).

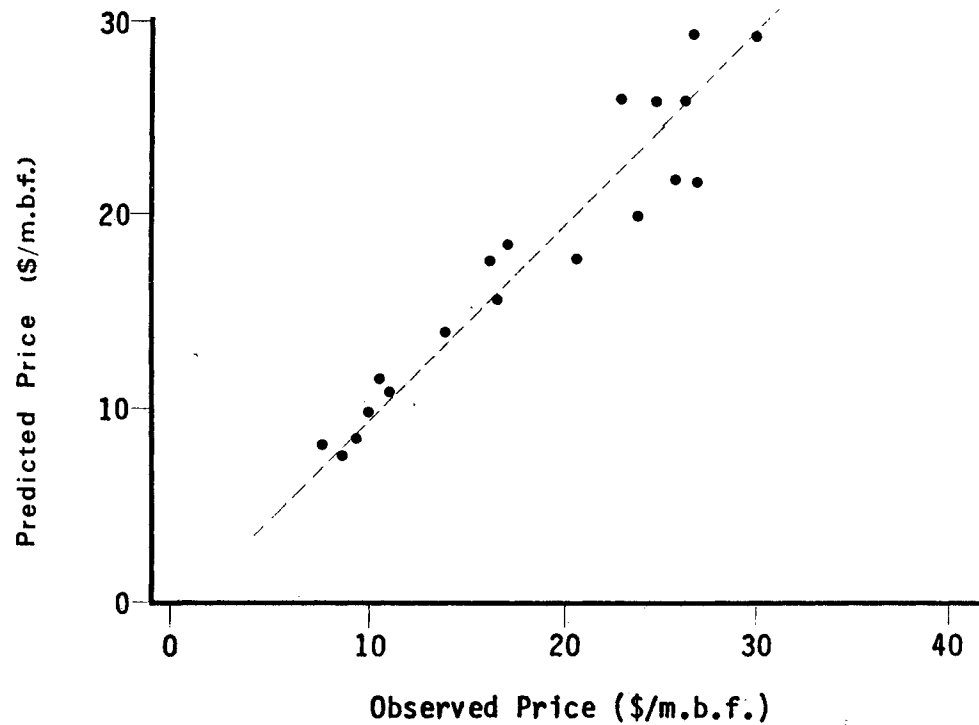
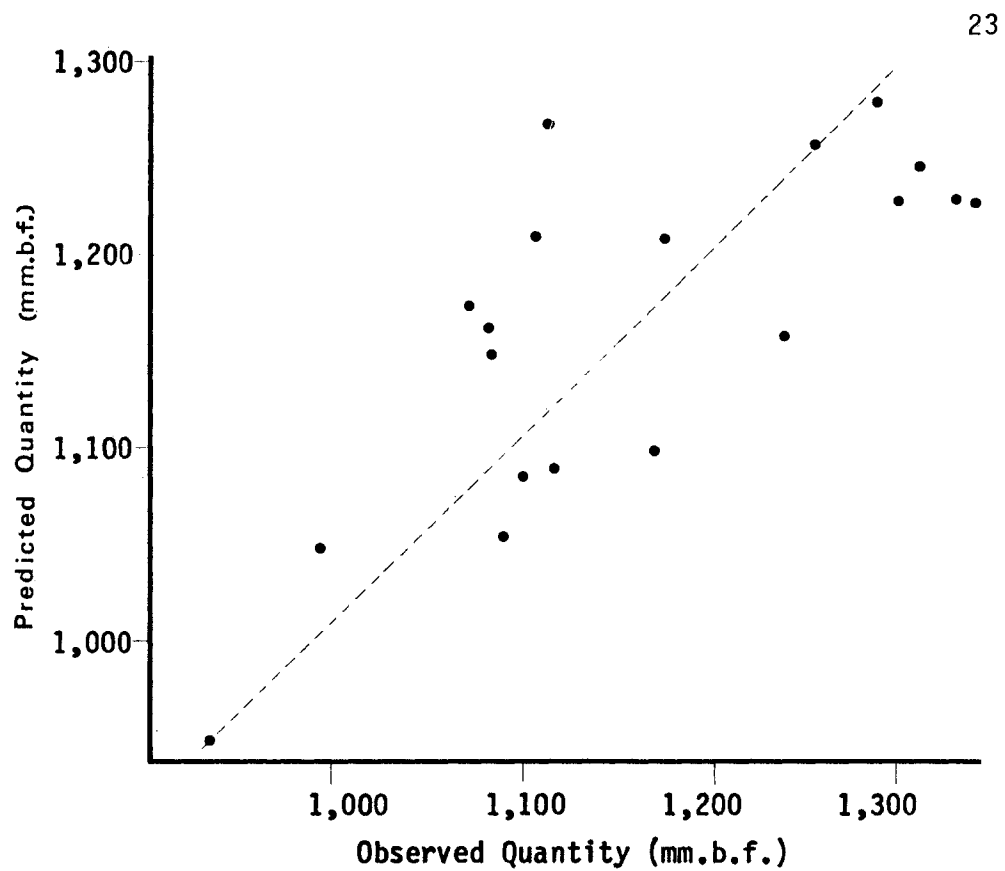


Figure 5.--Predicted and observed values; Montana demand and supply model

Table 4. Derived demand equations for individual National Forests in Montana and standard errors.

National Forest	Coefficients for-- ¹					Standard Error	Mean Harvest	Percent Standard Error
	Ps	Pf	Pc	S	Constant			
Beaverhead	-15,748	7,264	-5,574	-24.21	2,341,590	41,097	15,819	260
Bitterroot	-15,538	7,264	-5,574	-23.59	2,304,831	45,310	46,297	98
Custer	-15,846	7,264	-5,574	-24.50	2,358,745	41,075	1,958	2,098
Deerlodge	-15,702	7,264	-5,574	-24.07	2,333,518	42,378	22,233	191
Flathead	-14,939	7,264	-5,574	-21.81	2,200,320	48,044	131,015	37
Gallatin	-15,696	7,264	-5,574	-24.06	2,332,509	42,707	23,918	179
Helena	-15,733	7,264	-5,574	-24.17	2,338,996	40,365	18,353	220
Kootenai	-14,579	7,264	-5,574	-20.73	2,137,326	54,718	181,185	30
Lewis & Clark	-15,750	7,264	-5,574	-24.22	2,342,023	42,240	15,705	269
Lolo	-14,981	7,264	-5,574	-21.93	2,207,672	56,853	125,711	45

¹ Coefficients were computed using the sample period mean market shares.

Table 5. Mean market shares, percent standard errors and price elasticities of demand for National Forests in Montana (computed at the sample period means).

National Forest	Mean Market Share	Percent Standard Error	Price Elasticity of Demand
Kootenai	.1552	30	- 1.46
Flathead	.1115	37	- 2.06
Lolo	.1064	45	- 2.15
Bitterroot	.0390	98	- 6.08
Gallatin	.0198	179	- 11.88
Deerlodge	.0191	191	- 12.82
Helena	.0153	220	- 15.53
Beaverhead	.0135	260	- 18.07
Lewis and Clark	.0132	269	- 18.19
Custer	.0016	2,098	-146.85

The disaggregation procedure yields prodigious errors for the derived demand curves when the relative market shares are less than 0.10.

If a smaller geographical market area is used in the estimation of the large area demand and supply model, the errors for the derived demand curves may be less. Three National Forests, the Flathead, Kootenai and Lolo, have the lowest percent standard errors associated with the derived demand curves. These three forests comprised 32 percent of the stumpage market in Montana during 1980.

The price elasticities of demand for the individual National Forests range from -1.46 for the Kootenai to -146.85 for the Custer (see Table 5). As the market share declines the price elasticity of demand becomes more elastic. The greater the market share, the more responsive harvest levels are to changes in the stumpage price.

The lumber market is characterized by little product differentiation, prices determined in the market, few barriers to entry and an unconcentrated industry (Mead 1966). The lumber market is competitive and the demand curve facing individual firms is highly elastic while the market demand curve for the industry is inelastic. The greater the price elasticity of demand for the end product, the greater the elasticity for the input used to produce that product (Mansfield 1970). Since the firm's demand curve for wood products is highly elastic we would expect the firm's demand for timber to be elastic. As a timber supplier, an individual National Forest provides stumpage to a limited number of firms making up the industry. The price elasticity of demand for individual timber suppliers will depend on their market share or influence on price.

The same disaggregation procedure was used to obtain derived demand curves for the National Forest system and the westside and eastside National Forests in Montana.¹ The results are shown in

¹ The eastside forests in this analysis are the Gallatin, Deerlodge, Helena, Beaverhead, Lewis and Clark and Custer with the remaining forests being the westside forests.

Table 6 along with the standard errors. The percent standard errors for the National Forest system, westside forests and eastside forests are 20, 21 and 51 percent, respectively. Since the market shares for the National Forest system and westside forests are relatively greater than the individual forests, the slopes of the derived demand curves are more negative and the variance is less.

The price elasticity of demand for the National Forest system is -0.33. The National Forest system as a whole has greater influence on the price of stumpage than the individual National Forests. This influence on stumpage price declines as the market share drops. The price elasticities of demand for the westside and eastside forests are -0.42 and -2.80, respectively. The respective market shares are 0.44 and 0.08 (see Table 7).

Table 6. Derived demand equations for the eastside and westside National Forests and the National Forest system in Montana.

Aggregation	Coefficients for-- ¹					Standard Error	Mean Harvest	Percent Standard Error
	Ps	Pf	Pc	S	Constant			
Eastside	-15,179	7,264	-5,574	-22.52	2,242,125	49,796	97,985	51
Westside	-12,253	7,264	-5,574	-13.82	1,731,102	103,988	515,335	20
National Forest System	-11,573	7,264	-5,574	-11.79	1,612,176	126,979	613,320	21

¹ Coefficients were computed using the sample period mean market shares.

Table 7. Mean market shares, percent standard errors and price elasticities of demand for the eastside forests, westside forests and National Forest system in Montana (computed at the sample period means).

Area	Mean Market Share ^a	Percent Standard Error	Price Elasticity of Demand
Eastside	.0825	51	-2.80
Westside	.4370	20	-0.42
National Forest System	.5195	21	-0.33

^a The total mean market share for the National Forest system does not agree with the individual National Forests listed in Table 6. This is due to the changes in the administration of the Kaniksu and Coeur d'Alene National Forests.

Chapter 5

DISCUSSION

The results of this analysis indicate that the price elasticity of demand for timber in Montana is inelastic. The demand for Forest Service timber as a whole is price inelastic while for the individual National Forests it is price elastic. This indicates that the Forest Service has some influence on the price of stumpage in the market and the demand curve is not horizontal. The use of demand schedules brings the analysis of land allocation and harvest scheduling into a more realistic light.

During the National Forest system planning process every National Forest completes a plan supported by national and regional goals. Each National Forest must plan independently from other units of the National Forest system when determining timber output levels. A National Forest chooses the appropriate timber output level with the implicit assumption that the output from other suppliers is fixed or will not change when their own output level is altered. This is the same assumption Cournot (1838) made in his model of an oligopoly. Although there has been criticism of this assumption it is a factor in the Forest Service planning process. The timber demand and supply market resembles an oligopoly following the Cournot model. An oligopoly is a market situation where there is more than one seller but not so many that the activities of any or all do not have an effect on other producers (Leftwich 1979).

We can examine the changes in output levels using the Cournot model assuming the objective of timber suppliers is to maximize total revenue from sales. The price elasticity of demand for the National Forest system is inelastic while it is elastic for the individual National Forests. Taken as a whole, if the National Forest system decreases cut, total revenue would increase. However, this conflicts with the individual National Forests. One National Forest acting independently would increase revenue by increasing harvest. This increase in cut will subsequently lower the market price, thus (*ceteris paribus*) lowering the total revenue received by other suppliers. In response, other suppliers would adjust their own level of output to maximize revenue. Cournot saw these changes in output resulting in temporary benefits and the successive reactions of sellers as an unstable condition. Since one producer's change will force another to adopt a new output level the first producer would be punished (Cournot 1838). The equilibrium level of output is a function of the number of suppliers, in this case public and private, and the market demand curve. This output level is greater than the output level with a monopoly, but less than with a purely competitive industry.

From 1962 to 1980 there has been a 15 percent decrease in the volume of sawtimber on commercial timberland and a 19 percent decrease in acres of commercial timberland in Montana. Changes in growing stock have an influence on price, harvest and investment decisions. If stock changes, the effect on total revenue will be determined by the price elasticity of demand. With the Montana demand and supply model, if stock is decreased by one percent, there would be an eight

percent increase in price a one percent decrease in quantity and a seven percent increase in total revenue.¹ If stock is increased by one percent, there would be an opposite effect on price and quantity with a subsequent seven percent decrease in total revenue. The distribution of the changes in quantity and total revenue is dependent on which ownerships change stock. If the National Forest system reduced stock, total revenue would increase for the Forest Service but decrease for that particular National Forest whose stock was reduced. The long run effect will be an adjustment in the rotation through price. If price increases, the rotation will decrease assuming time is a substitute for other decision variables. If time is not a substitute but a complement, the opposite effect on rotation may occur. (Jackson 1980).

This study has shown that an econometric model can be used to estimate the demand and supply for timber in a relatively small geographical area. Due to the presence of multicollinearity in the model, strong conclusions regarding the parameters cannot be made. However, the signs of the coefficients are correct, significant and the elasticities are reasonable. Further analysis could be made with ridge regression to correct for multicollinearity but that was beyond the scope of this study. The problem encountered when using a large

¹ Percent changes were calculated using the sample period mean values.

area supply and demand relationship to estimate the demand curve for a smaller area is evident by the errors of the analytically derived equations. The standard errors are high for the individual National Forest demand equations, but the question remains whether this error is greater or lesser than the error associated when using a horizontal demand curve.

APPENDIX A

Input Demand Function

From economic theory the objective of the firm is to maximize profit. Profit equal to total revenue (price times quantity of output) less total costs (factor prices times input quantities). The factors of production in the manufacture of wood products are stumpage, labor and capital. The production process is from stump to final product. The input demand function for a factor of production can be obtained from the necessary conditions for profit maximization. The profit maximizing objective function of the competitive firm is as follows:

$$\text{Max } \pi = pf(s, l, k) - (w_1s + w_2l + w_3k) \quad (1)$$

where π = profit
 p = price of the end product
 s = quantity of stumpage
 l = quantity of labor
 k = quantity of capital
 w_i = factor prices, $i = 1, 2, 3$.

The first-order maximization conditions are as follows:

$$\frac{\partial \pi}{\partial s} = p \frac{f(s, l, k)}{s} - w_1 \quad \text{set} = 0 \quad (2)$$

$$\frac{\partial \pi}{\partial l} = p \frac{f(s, l, k)}{l} - w_2 \quad \text{set} = 0 \quad (3)$$

$$\frac{\partial \pi}{\partial k} = p \frac{f(s, l, k)}{k} - w_3 \stackrel{\text{set}}{=} 0 \quad (4)$$

Profit maximization requires that the factor price of each input be equated to the value of the marginal product of that input (Russel et al. 1979). The implicit input demand function for stumpage is derived by solving the above system of equations for stumpage, s , as a function of the prices:

$$s = s^*(p, w_1, w_2, w_3) \quad (5)$$

The demand for stumpage is a function of the price of the end product, price of stumpage and the costs of labor and capital in the logging and manufacturing sectors. The input demand function used in this analysis is as follows:

$$Q_d = f(P_s, P_f, P_c)$$

where Q_d = the quantity demanded

P_s = the price of stumpage

P_f = the price of the end product

P_c = the logging, hauling and manufacturing costs (the costs of labor and capital).

APPENDIX B

Data Used in the Demand and Supply Model

TABLE 8

MONTANA DATA

<u>Year</u>	<u>Harvest</u> m.b.f.	<u>P_s^a</u> \$/m.b.f.	<u>P_f^b</u>	<u>P_c^a</u> \$/m.b.f.	<u>Stock</u> mm.b.f.
1962	1,116,900	7.51	102.59	59.37	107,802
1963	1,293,200	7.80	103.20	58.55	108,267
1964	1,259,500	8.77	103.67	61.69	107,233
1965	1,315,400	9.44	101.96	60.12	106,199
1966	1,339,200	10.75	103.15	62.80	105,165
1967	1,177,600	10.18	100.00	61.34	104,131
1968	1,339,400	16.92	108.42	60.86	103,097
1969	1,302,373	23.96	114.08	65.69	102,064
1970	1,093,313	13.45	98.34	67.30	101,030
1971	1,243,450	20.22	104.51	67.99	100,123
1972	1,083,600	26.46	114.02	72.43	99,216
1973	1,117,423	26.23	132.34	78.87	98,309
1974	1,088,283	22.77	125.04	82.55	97,402
1975	1,008,700	16.40	109.93	97.15	96,495
1976	1,106,144	26.33	121.50	97.50	95,588
1977	1,124,577	24.77	131.77	103.79	94,681
1978	1,171,420	29.56	143.01	113.17	93,774
1979	1,095,429	26.97	143.47	125.65	92,867
1980	944,802	16.18	128.70	133.97	91,960
Mean	1,169,511	18.15	115.25	80.57	100,284

^a All dollar values are expressed in constant 1967 dollars, adjusted using the Implicit Price Deflator for the Gross National Product.

^b The lumber and wood products price index is adjusted using the GNP Implicit Price Deflator.

Description and Source of Data

Harvest Volume: Q_d and Q_s

Description: Annual volume of timber cut from commercial timberland in Montana for all ownerships, m.b.f./yr. (log scale).

Source: Schuster, Ervin G. 1978. Montana's timber harvest and timber-using industry: a study of relationships. Montana Conserv. Exp. Stn., Bull. 41. Missoula, MT. 36 p.

U.S.D.A. Forest Service. 1981. Montana timber harvest. Region 1, Missoula, MT.

Stumpage Price: Ps

Description: Annual National Forest cut value in dollars per m.b.f. (log scale).

Source: U.S.D.A. Forest Service. Quarterly cut and sold reports. Timber management files, Region 1, Missoula, MT.

Lumber and Wood Products Price Index: Pf

Description: Annual lumber and wood products price index.

Source: Various publications of the Survey of Current Business as reported by the U.S. Dept. of Labor, Bureau of Labor Statistics.

Logging, Hauling and Manufacturing Costs: Pc

Description: Annual logging, hauling and manufacturing costs for Forest Service sales greater than two thousand dollars as recorded on the timber appraisal form, dollars per m.b.f. (log scale).

Source: Calendar and quarterly summaries of the timber appraisal forms. Timber management files, Region 1, Missoula, MT.

Stock-Inventory: S

Description: Annual volume of softwood and hardwood sawtimber on commercial forest land in Montana for all ownerships, mm.b.f. (log scale). Volumes were converted from International 1/4 inch log rule to Scribner log rule using a ratio of 0.97275 Scribner per International 1/4 inch log rule. The conversion factor was calculated using data from Scribner and International 1/4 inch volume tables for 16 foot logs and dbh classes 6 to 60.

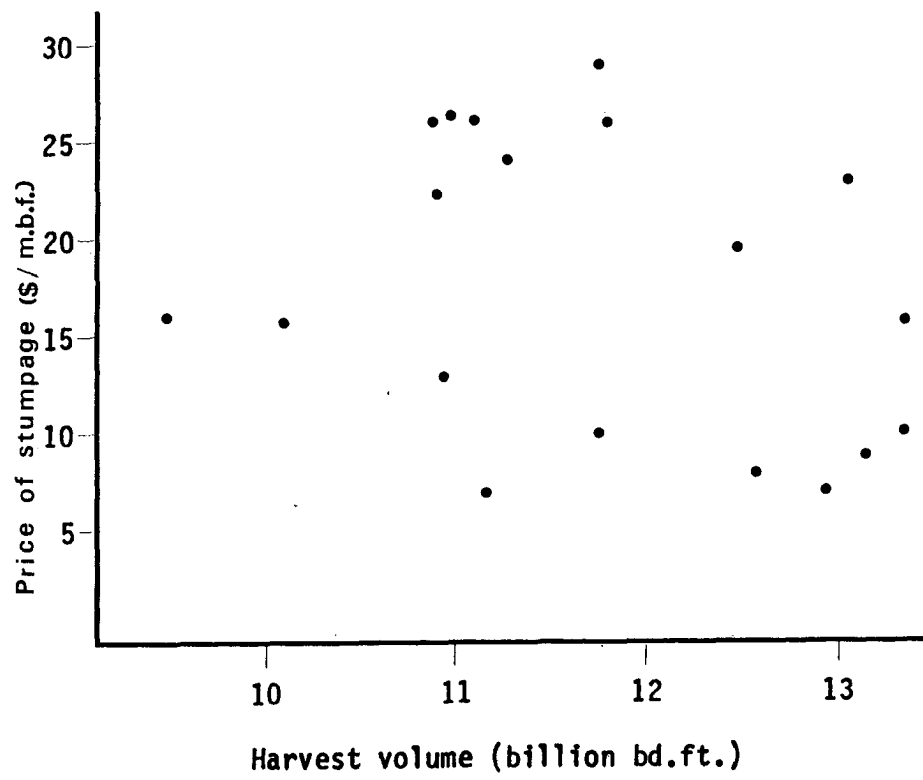
Source: U.S.D.A. Forest Service. 1973. The outlook for timber in the United States. Forest Service Rept. No. 20. 367 p.

U.S.D.A. Forest Service. 1978. Forest Statistics of the U.S., 1977. Washington, D.C. 133 p.

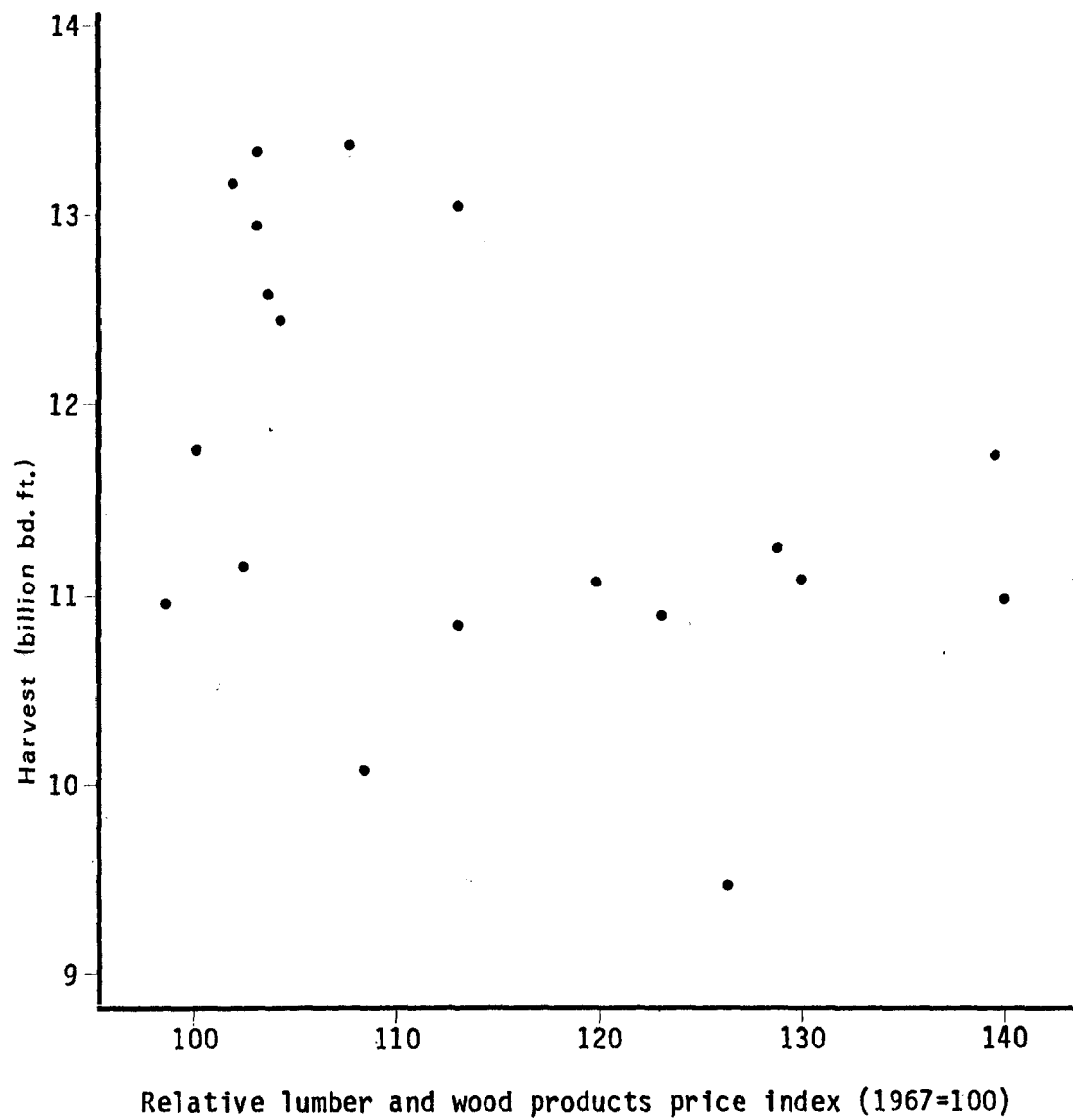
Following are graphs of the dependent variable, Q_d , plotted with the endogenous variable P_s and the exogenous variables P_f , P_c and S .

Graph 1

Price of Stumpage and Harvest

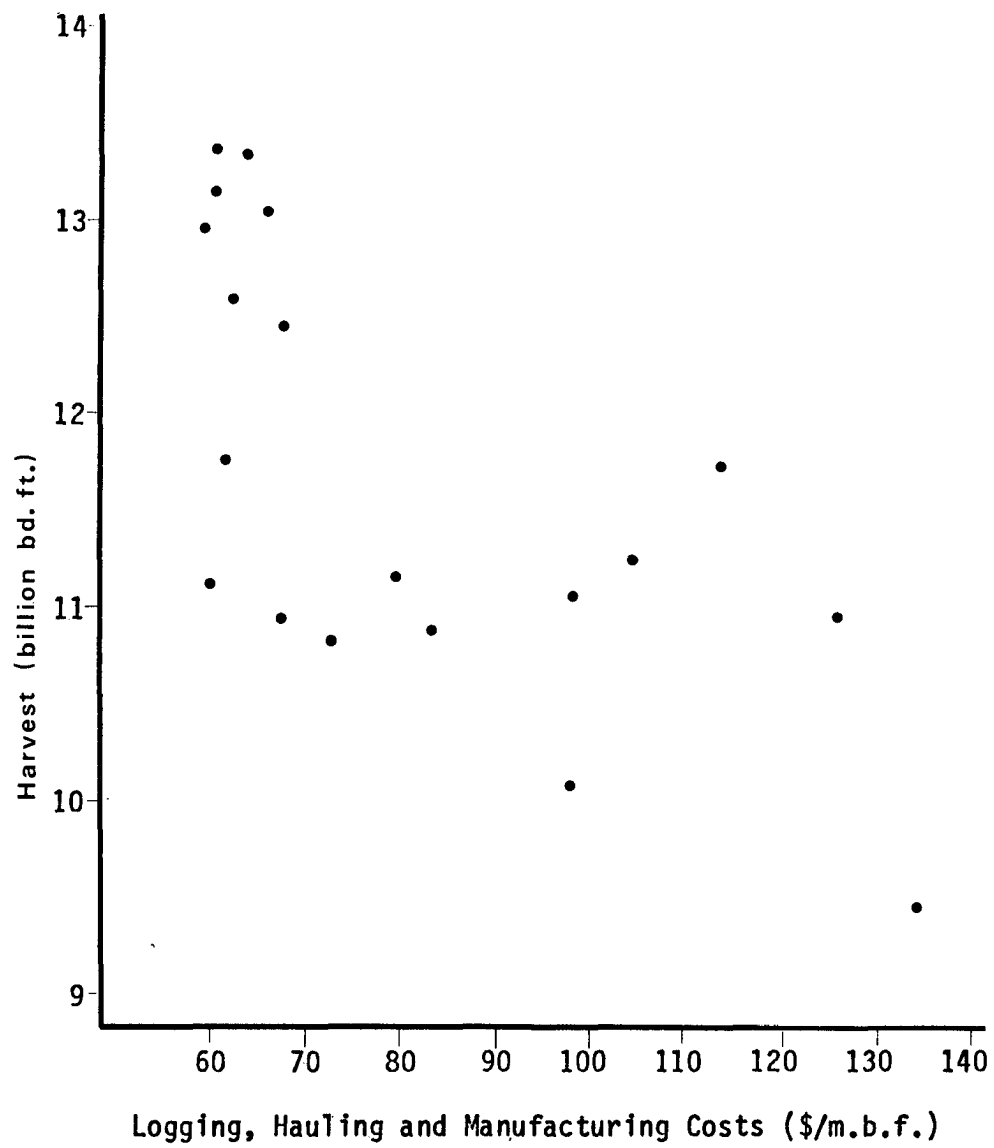


Graph 2

Harvest and Relative Lumber and
Wood Products Price Index

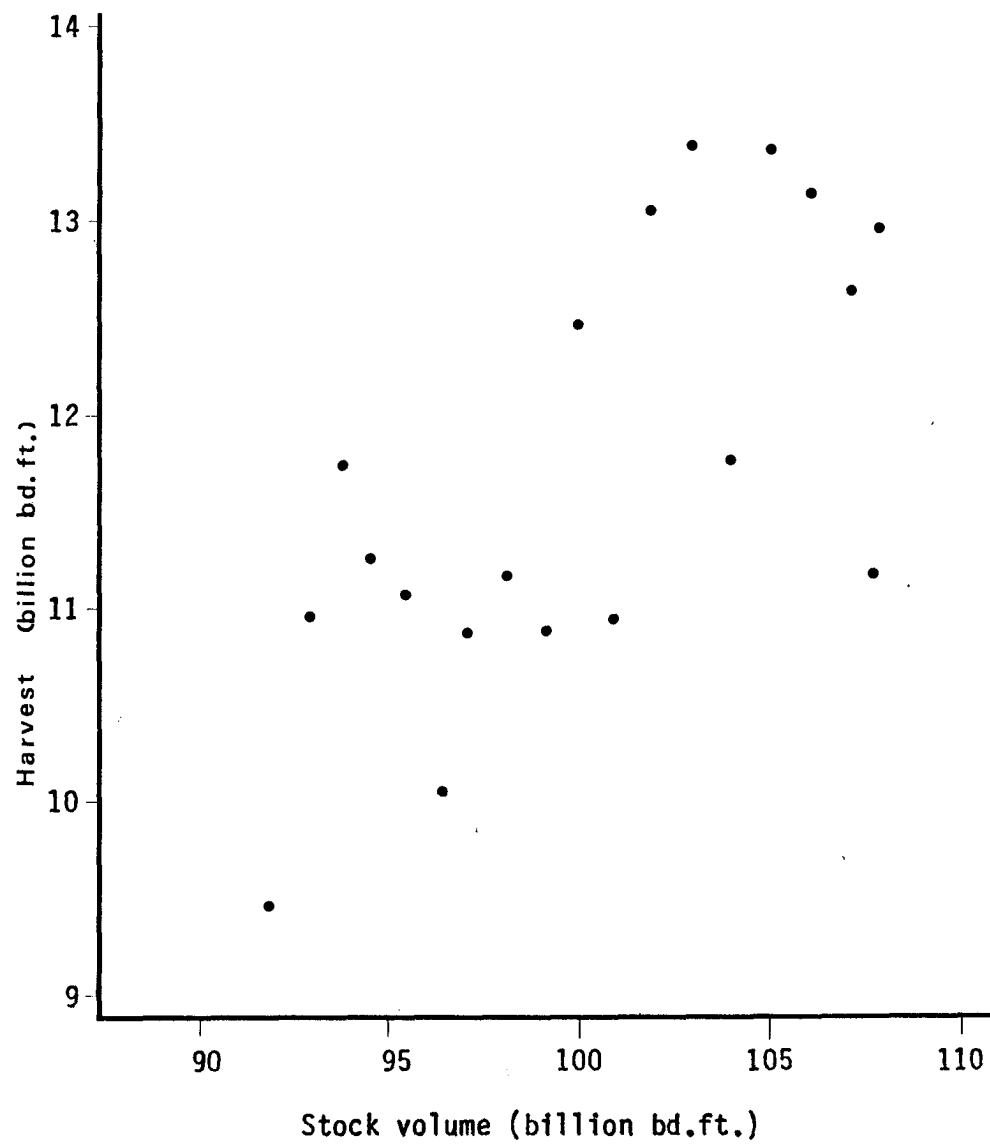
Graph 3

Harvest and Logging, Hauling and
Manufacturing Costs



Graph 4

Harvest and Stock



APPENDIX C

Variance of a National Forest's Demand Equation

The model for the derived demand equation is:

$$q_{dd_t} = Qd_t - (1-k)Qs_t \quad (1)$$

where k = an individual forest's market share
 q_{dd_t} = derived demand for an individual forest's
timber in time t , m.b.f./yr
 Qd_t = large area demand equation
 Qs_t = large area supply equation.

In equilibrium an individual forest's derived demand is equal to the harvest or cut for a point in time. In this study the time interval is one year. The annual harvests from individual National Forests are equal to their derived demands at the prevailing prices or are points of equilibrium of supply and demand. The estimated annual derived demand is as follows:

$$\hat{q}_{dd_t} = \hat{Qd}_t - (1-k)\hat{Qs}_t \quad (2)$$

where $\hat{}$ indicates the predicted variable.

The variance is a measure of the dispersion of the observed values of the dependent variable around their predicted values (Koutsoyiannis 1979). The estimated variance of the derived demand equation is a measure of the dispersion of the values of q_{dd} around

their predicted values \hat{q}_{dd} . The formula for the variance is as follows:

$$\text{Var}(q_{dd}) = \frac{\sum (q_{dd_t} - \hat{q}_{dd_t})^2}{n-d} \quad (3)$$

$$= \frac{\sum [(Qd_t - (1-k_t)Qs_t) - (\hat{Q}d_t - (1-\hat{k}_t)\hat{Q}s_t)]^2}{n-d} \quad (4)$$

where $n-d$ = degrees of freedom.¹

If we let $m_t = (1-k_t)$, then equation (4) can be rewritten as:

$$\text{Var}(q_{dd}) = \frac{\sum [(Qd_t - m_t Qs_t) - (\hat{Q}d_t - \hat{m}_t \hat{Q}s_t)]^2}{n-d} \quad (5)$$

$$= \frac{\sum [(Qd_t - \hat{Q}d_t)^2 + (m_t Qs_t - \hat{m}_t \hat{Q}s_t)^2 - 2 (Qd_t - \hat{Q}d_t)(m_t Qs_t - \hat{m}_t \hat{Q}s_t)]}{n-d} \quad (6)$$

The formula for the variance of a derived equation is equation (6).

Empirically each component is calculated separately. Equation (6) can be separated into the following three parts:

- 1) $\frac{\sum (Qd_t - \hat{Q}d_t)^2}{n-d}$ = the variance of the market demand
- 2) $\frac{\sum (m_t Qs_t - \hat{m}_t \hat{Q}s_t)^2}{n-d}$ = the variance of the non-national forest's supply
- 3) $\frac{\sum 2[(Qd_t - \hat{Q}d_t)(m_t Qs_t - \hat{m}_t \hat{Q}s_t)]}{n-d}$ = two times the covariance between the market demand and non-national forest's supply.

¹ There are 14 degrees of freedom (19-5=14). The derived demand equation consists of five parameters (see Appendix F).

APPENDIX D

Identification of the Demand and Supply Model

A model is identified if it is in a unique statistical form, enabling unique estimates of its parameters (Koutsoyiannis 1979). The model is as follows:

$$Q_d = f(P_s, P_f, P_c) \quad (1)$$

$$Q_s = f(P_s, S) \quad (2)$$

$$Q_d = Q_s. \quad (3)$$

There are three equations, three endogenous variables (Q_d , Q_s , P_s) and three exogenous variables (P_f , P_c , S). The system is complete since there are as many equations as endogenous variables. Two conditions must be satisfied for an equation to be identified. The first is the order condition which is necessary but not sufficient for identification.

For an equation to be identified the total number of variables excluded from it but included in other equations must be at least as great as the number of equations of the system less one.
(Koutsoyiannis 1979)

The following condition must be satisfied:

$$(K-M) \geq (G-1)$$

where K = number of total variables in the model

M = number of variables in a particular equation

G = total number of equations.

The second condition is the rank condition.

In a system of G equations any particular equation is identified if and only if it is possible to construct at least one non-zero determinant of order $(G-1)$ from the coefficients of the variables excluded from that particular equation but contained in the other equations of the model.
(Koutsoyiannis 1979)

Identification of the Demand Function

1) Order condition $(K-M) \geq (G-1)$

$$K=6, M=4, G=3$$

$$2=2$$

2) Rank condition

Table of Structural Parameters

Equations	Variables					
	Qd	Ps	Pf	Pc	Qs	S
1st equation	-1	a1	a2	a3	0	0
2nd equation	0	b1	0	0	-1	b2
3rd equation	-1	0	0	0	-1	0

Table of Parameters of Variables
Excluded from the Demand Equation

	Qs	S
2nd equation	-1	b2
3rd equation	-1	0

From the above table we can form a non-zero determinant of order

$$(G-1) = (3-1) = 2:$$

$$\begin{vmatrix} -1 & b2 \\ -1 & 0 \end{vmatrix} = b2.$$

Both the order and rank conditions are satisfied, therefore, the first equation of the model is identified. Since the equality holds for the order condition, the demand equation is exactly identified.

Identification of the Supply Function

1) Order condition $(K-M) \geq (G-1)$

$$K=6, M=3, G=3$$

$$3 \geq 2$$

2) Rank condition

Table of Parameters of Variables
Excluded from the Supply Equation

	Qd	Pf	Pc
1st equation	-1	a2	a3
3rd equation	-1	0	0

From the above table we can form a non-zero determinant of order

$$(G-1) = (3-1) = 2:$$

$$\begin{vmatrix} -1 & a2 \\ -1 & 0 \end{vmatrix} = a2.$$

Both the order and rank conditions are satisfied, therefore, the second equation of the model is identified. Since the inequality holds for the order condition, the supply equation is overidentified.

APPENDIX E

Correlation coefficients and the Farrar-Glauber test for the presence of multicollinearity

TABLE 9

CORRELATION COEFFICIENTS

	Qds	Ps	Pf	Pc
Ps	-.336486			
Pf	-.448624	.707012		
Pc	.683251	.451895	.851294	
S	.692752	-.705305	-.830357	-.914621

Farrar-Glauber Test for Multicollinearity

Multicollinearity may be considered a departure from orthogonality. Glauber and Farrar suggest a chi-squared, $*X^2$, test for detecting the strength of multicollinearity. The hypotheses are as follows:

Ho: the independent variables are orthogonal

Ha: the independent variables are not orthogonal

The empirical $*X^2$ is obtained by the following formula:

$$*X^2 = -[n-1-1/6(2k+5)] \ln \left| \frac{\text{Standardized}}{\text{determinant}} \right| \quad (1)$$

where $*X^2$ = the empirical chi-squared

n = sample size

k = number of explanatory variables.

The empirical value is compared with the theoretical value at the chosen level of significance with $v = 1/2k (k-1)$ degrees of freedom.

If the observed value is greater than the theoretical value, we reject the assumption of orthogonality.

Test for Multicollinearity in the Demand Equation

The standardized determinant is as follows:

	Ps	Pf	Pc
Ps	1	.707012	.451895
Pf	.707012	1	.851294
Pc	.451895	.851294	1

The empirical chi-squared statistic is

$$\begin{aligned} * \chi^2 &= -[19-1-1/6(2(3)+5)] \ln 0.1151921 \\ &= 34.94 \end{aligned} \quad (2)$$

The theoretical value at the 90 percent confidence level is 6.25, therefore, there is multicollinearity in the demand equation.

Test for Multicollinearity in the Supply Equation

The standardized determinant is as follows:

	Ps	S
Ps	1	-.705305
S	-.705305	1

The empirical chi-squared statistic is

$$\begin{aligned} * \chi^2 &= -[19-1-1/6(2(2)+5)] \ln 0.5025449 \\ &= 11.35 \end{aligned} \quad (3)$$

The theoretical value at the 90 percent confidence level is 2.71, therefore, there is multicollinearity in the supply equation.

APPENDIX F

Calculation of Derived Demand Equations

The individual National Forest demand equations are analytically derived from the Montana demand and supply equations. Jackson's (1981) derived demand model is used for the disaggregation to the National Forest level. The large area demand and supply equations are as follows:

$$Q_d = 919,521 - 7,607.53P_s + 7,263.74P_f - 5,574.23P_c \quad (1)$$

$$Q_s = -1,441,530 + 8,251.79P_s + 24.5433S \quad (2)$$

The derived demand model is

$$q_{dd} = Q_d - \bar{m}Q_s \quad (3)$$

where $\bar{m} = (1-\bar{k})$ or the mean market share of all other ownerships.

Substituting equations (1) and (2) for Q_d and Q_s in equation (3) yields the derived demand equation:

$$q_{dd} = (919,521 + 1,441,530\bar{m}) - (7,607.53 + 8,251.79\bar{m})P_s + 7,263.74P_f - 5,574.23P_c - 24.5433\bar{m}S \quad (4)$$

The sample period mean market shares were substituted for k , $m = (1-k)$, to arrive at the individual National Forest demand equations. Each equation consists of seven coefficients plus the market share, m . Using the mean market share reduces the equation to five parameters.

SELECTED BIBLIOGRAPHY

- Adams, D. M. 1975. The response of softwood stumpage and end product prices to changes in National Forest timber harvest schedules. Report for a Coop Aid Agreement with the Pac. Northwest For. and Range Exp. Stn., Oregon State University, School of Forestry, Mimeograph. 74 p.
- _____ and R. W. Haynes. 1980. The 1980 softwood timber assessment market model: structure, projections, and policy simulations. For. Sci. Monograph 22. 64 p.
- Bronfenbrenner, M. 1961. Notes on the elasticity of derived demand. Oxford Economic Papers. 13:254-261.
- Connaughton, K. P. 1981. Current approaches to modeling the demand for National Forest stumpage. Unpublished paper on file at the USFS Pacific Northwest For. and Range Exp. Sta., Portland, OR. 66 p.
- Cournot, A. 1838. Researches into the mathematical principles of the theory of wealth. MacMillan Company, New York. 209 p.
- Hamilton, T. E. 1970. Stumpage price responses to changes in volume of timber sold. USDA For. Serv. Res. Note PSW-315. Pac. Southwest For. and Range Exp. Stn., Berkeley, CA. 6 p.
- Haynes, R. W. 1977. A derived demand approach to estimating the linkage between stumpage and lumber markets. For. Sci. 23:281-288.
- _____ and D. M. Adams. 1979. Possible changes in regional forest product output and consumption during the next 50 years. For. Products Journal. 29(10):75-80.
- _____, K. P. Connaughton and D. M. Adams. 1981. Projections of the demand for National Forest stumpage by region; 1980-2030. USDA For. Serv. Res. Paper PNW-282. Pac. Northwest For. and Range Exp. Stn., Portland, OR. 13 p.
- _____, K. P. Connaughton and D. M. Adams. 1980. Stumpage price projections for selected western species. USDA For. Serv. Res. Note PNW-367. Pac. Northwest For. and Range Exp. Stn., Portland, OR. 14 p.
- Hrubes, R. J. and D. I. Navon. 1976. Application of linear programming to downward sloping demand problems in timber production. USDA For. Serv. Res. Note PSW-315. Pac. Southwest For. & Range Exp. Stn., Berkeley, CA. 6 p.
- Jackson, D. H. 1981. Sub-regional timber demand analysis: remarks and an approach for prediction. Unpublished. 14 p.
- _____. 1980. The microeconomics of the timber industry. Westview Press, Inc., Boulder, CO. 13 p.

- Josephson, H. R. 1976. Economics and National Forest timber harvests. *J. For.* 74:605-608.
- Leftwich, R. H. 1979. The price system and resource allocation. The Dryden Press, Hinsdale, Illinois. 487 p.
- Mansfield, E. 1970. Microeconomics theory and applications. W. W. Norton and Company, Inc., New York. 478 p.
- Mead, W. J. 1966. Competition and oligopsony in the Douglas fir lumber industry. University of California Press, Berkeley, CA. 276 p.
- Nautiyal, J. C. and K. S. Fowler. 1980. Optimum forest rotation in an imperfect stumpage market. *Land economics*. 56(2):213-226.
- Navon, D. I. 1971. Timber RAM... a long-range planning method for commercial timber lands under multiple-use management. USDA For. Serv. Res. Paper PSW-70. Pac. Southwest For. and Range Exp. Stn., Berkeley, CA. 22 p.
- Ott, L. 1977. An introduction to statistical methods and data analysis. Duxbury Press, Belmont, CA. 730 p.
- Rao, P. and R. L. Miller. 1971. Applied econometrics. Wadsworth Publishing Company, Inc., Belmont, CA. 325 p.
- Robinson, V. L. 1974. An econometric model of softwood lumber and stumpage markets, 1947-1967. *For. Sci.* 20:171-178.
- Russell, R. R. and M. Wilkinson. 1979. Microeconomics a synthesis of modern and neoclassical theory. John Wiley and Sons, New York. 459 p.
- Schreuder, G. F., T. R. Waggener and E. L. Medena. 1976. Estimation of local short run stumpage price and sale response to National Forest timber offering. Report in fulfillment of Coop Aid Agreement with the Pac. Northwest For. and Range Exp. Stn., University of Washington, College of Forest Resources, Mimeo. 153 p.
- Schuster, E. G. 1978. Montana's timber harvest and timber-using industry: a study of relationships. Montana Forest and Conservation Exp. Stn. Bull. 41. Missoula, MT. 36 p.
- U.S.D.A. Forest Service. 1978. Forest statistics of the U.S., 1977. Review draft, Washington, D.C. 133 p.
- _____. 1973. The outlook for timber in the United States. Forest Service rept. no. 20. 367 p.
- Walker, J. L. 1980. An analysis of timber demand in the Lolo National Forest market area. Unpublished. 32 p.